Linear Dependence and Independence

Definition: Vectors $v_1, v_2, \ldots, v_k$ are **linearly independent** if the linear combo problem

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the zero solution: $c_1 = 0, c_2 = 0, \ldots, c_k = 0$. That is, there is no non-trivial way to build the zero vector as a linear combo of $v_1, v_2, \ldots, v_k$. If there is a non-trivial way, i.e. at least one $c_i \neq 0$, then we say the vectors are **linearly dependent**.

$$\text{linearly dependent} \iff \begin{cases} \text{one vector can be written as} \\ \text{a linear combo of the others} \end{cases}$$

$$\iff \begin{cases} \text{that vector can be eliminated} \\ \text{from linear combos in favor of} \\ \text{all the others} \end{cases}$$

Geometrically in $\mathbb{R}^n$:

two vectors are linearly dependent if the lie on the same line
three vectors are linearly dependent if the lie on the same plane