The Uniqueness Theorem

Ex: $\mathbb{R}^2 = \text{Span}\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\} = \text{Span}\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\}$
Therefore the dimension of $\mathbb{R}^2$ is 2
Similarly the dimension of $\mathbb{R}^n$ is n (find a basis by looking at I)

Theorem: (Uniqueness of Linear Combos Relative to a Basis)
If v is in vector space V, and if

$$v = a_1 v_1 + \cdots + a_n v_n \text{ and } v = b_1 v_1 + \cdots + b_n v_n$$

where $v_1, ..., v_n$ is a basis of V, then

$$a_1 = b_1, ..., a_n = b_n$$

We refer to $a_1, ..., a_n$ as the coordinates of v relative to the basis.

Note: Different bases lead to different coordinates:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{x_1 + x_2}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{x_1 - x_2}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$