

Representations of the Null Space

In our equations, each of u, v, w, y corresponds to a column of the matrix. **Pivot variables** are those corresponding to columns with a pivot (circled). This means u and w . The remaining variables v and y are called **free variables**. By convention we solve our system for the pivot variables in terms of the free variables: $u = -3v + y, w = -y$. Thus

$$x = \begin{bmatrix} u \\ v \\ w \\ y \end{bmatrix} = \begin{bmatrix} -3v + y \\ v \\ -y \\ y \end{bmatrix} = v \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \left. \vphantom{\begin{bmatrix} u \\ v \\ w \\ y \end{bmatrix}} \right\} \begin{array}{l} \text{linear combo} \\ \text{representation} \\ \text{of } N(A) \end{array}$$

Compare this to

$$\left. \begin{array}{l} u + 3v + 3w + 2y = 0 \\ 2u + 6v + 9w + 7y = 0 \\ -u - 3v + 3w + 4y = 0 \end{array} \right\} \begin{array}{l} \text{restrictions} \\ \text{representation} \\ \text{of } N(A) \end{array}$$