Solving the Homogeneous System

Note: We have introduced the elimination steps

$$[A|b] \longrightarrow [U|c] \longrightarrow [R|d]$$

and so $Ax = b$, $Ux = c$, and $Rx = d$ have the same solutions. The form $[R|d]$ of the system is the easiest to solve.

Recall that the null space $N(A)$ is by definition characterized by the restrictions representation. We elaborate now on how to find the linear combo representation. The goal is to solve $Ax = 0$. Equivalently $Rx = 0$. That is,

$$\begin{bmatrix} \textcircled{1} & 3 & 0 & -1 \\ 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$