

Reduced Row Echelon Form

Reduced Row Echelon Form (RREF): After moving to the REF

- continue elimination to produce zeros 'above' each pivot
- divide each row by its pivot

$$\text{i.e. } A \longrightarrow \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{REF}} \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R \xrightarrow{\text{RREF}}$$

Theorem: For each $m \times n$ matrix A there is an $m \times m$ permutation matrix P , a lower triangular $m \times m$ matrix L (1's on the diagonal) and an $m \times n$ upper triangular matrix U (the row echelon form of A) such that

$$PA = LU$$