A Generalization of Gaussian Elimination

We now need to generalize Gaussian elimination to matrices that are not necessarily square.

$$A = \begin{bmatrix} \textcircled{1} & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix} \xrightarrow[R_3 + R_1]{R_2 - 2R_1} \begin{bmatrix} \textcircled{1} & 3 & 3 & 2 \\ 0 & 0 & \textcircled{3} & 3 \\ 0 & 0 & 6 & 6 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_2} \begin{bmatrix} \textcircled{1} & 3 & 3 & 2 \\ 0 & 0 & \textcircled{3} & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The **row echelon form** U of A
Note the step-like structure
of this matrix

Row Echelon Form (REF): After elimination (allowing row exchanges)

- a pivot is the first non-zero entry in each non-zero row (circled)
- zeros appear below each pivot
- pivots move to the right as we move down the rows
- zero rows appear at the bottom