Example Continued

If instead we actually solve the system $Ax = 0$, we get

$$[A|0] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & 0 \end{array}\right] \xrightarrow{\text{G-E}} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

All we learn is that $x_2 = -x_1$. If we set $x_1 = c$, then $x_2 = -c$, and

$$x = \begin{bmatrix} c \\ -c \end{bmatrix} = c \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

This is the linear combo representation: $N(A)$ is the set of all linear combos (i.e. multiples) of the special vector (and solution)

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$