Example Continued

If instead we attempt to solve $Ax = b$ with a general b, we get

$$[A|b] = \left[\begin{array}{cc|c} 1 & 1 & b_1 \\ 2 & 2 & b_2 \end{array}\right] \xrightarrow{\text{G-E}} \left[\begin{array}{cc|c} 1 & 1 & b_1 \\ 0 & 0 & b_2 - 2b_1 \end{array}\right]$$

For the system to be consistent we need $b_2 - 2b_1 = 0$. This gives us the restrictions representation: $C(A)$ in this case is the set of all vectors b for which $b_2 - 2b_1 = 0$.

Now look at the null space $N(A)$. By its very definition it is characterized by the restrictions representation.
Ex: With A as above, the null space is the set of all vectors x whose entries satisfy $x_1 + x_2 = 0$ (since this is all $Ax = 0$ reduces to).