

Superposition Revisited

Ex:

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix} x = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix} \quad \leftarrow \text{a vector in } C(A)$$

i.e. $(5) - 2(5) + 5(1) = 0$

$$\left[\begin{array}{cccc|c} 1 & 3 & 3 & 2 & 1 \\ 2 & 6 & 9 & 7 & 5 \\ -1 & -3 & 3 & 4 & 5 \end{array} \right] \xrightarrow{\text{G-J}} \left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & -2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Thus $u = -2 - 3v + y$, $w = 1 - y$. Hence:

$$x = \begin{bmatrix} u \\ v \\ w \\ y \end{bmatrix} = \begin{bmatrix} -2 - 3v + y \\ v \\ 1 - y \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{x_p \text{ particular soln}} + v \underbrace{\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{x_h \text{ general soln of homogeneous}} + y \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$