

Resolving a Question

We have seen above that b is a vector in \mathbb{R}^3 that lies in the plane $b_3 - 2b_2 + 5b_1 = 0$. Usually a plane in \mathbb{R}^3 is the set of all linear combos of just two vectors, but in the linear combo representation of $C(A)$ we have four vectors. Can this observation be explained?

Note:

$$\begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

We conclude that any linear combo of the four vectors can also be written as a linear combo of the first and the third:

$$b = u \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + v \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix} \left. \vphantom{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}} \right\} \begin{array}{l} \text{alternate linear} \\ \text{combo representation} \\ \text{of } C(A) \end{array}$$