The Importance of the Column Space and Null Space

If the columns of A are $a_1, \ldots, a_n$, then

$$b = Ax = \begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 a_1 + \cdots + x_n a_n$$

and so the column space is also the space of all b's for which $Ax = b$ is consistent. This answers Q1.

Let x_p be one solution of $Ax = b$. If there are infinitely many solutions, just pick one at random. Now let x be any other solution. Then

$$A(x - x_p) = Ax - Ax_p = b - b = 0$$

and so $x_n = x - x_p$ is in the null space of A. This gives superposition:

$$x = x_p + x_n$$

The general solution is a particular solution plus the general solution x_n of the homogeneous system. The size of $N(A)$ (which contains all x_n's) measures the degree of non-uniqueness. This answers Q2.