

Column Space and Null Space

Let us return to looking at a linear system of m equations in n unknowns:

$$\underbrace{A}_{\substack{\text{an } m \times n \\ \text{matrix}}} \underbrace{x}_{\text{in } \mathbb{R}^n} = \underbrace{b}_{\text{in } \mathbb{R}^m}$$

Q1: What is the set of b 's giving consistency? (Existence of solutions)

Q2: What is the structure of the set of solutions? (Unique or infinitely many)

The answers lie in two important subspaces:

Definition: The **column space** $C(A)$ of A is the set of all linear combinations of the columns of A .

Definition: The **null space** $N(A)$ of A is the set of all solutions of the homogeneous system $Ax = 0$.