

Example of a Non-subspace

Ex: $V = \mathbb{R}^2$, S is the first quadrant, i.e. the subset of all vectors whose entries are non-negative.

Closure under addition is OK: with $a \geq 0, b \geq 0, c \geq 0, d \geq 0$

$$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$$

which is again in S since $a+c \geq 0$ and $b+d \geq 0$.

Closure under scalar multiplication fails: with $a \geq 0$ and $b \geq 0$

$$-1 \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -a \\ -b \end{bmatrix}$$

which is NOT in S since $-a \leq 0$ and $-b \leq 0$. This is not a subspace!