Subspaces and Closure Rules

For us the most important vector spaces will be subsets of vector spaces we already have.

Definition: Let S be a subset of a vector space V. Then we say that S is a **subspace** of V if S is also a vector space (using the operations defined for V). We need linear combinations of vectors in S to result in new vectors **also in** S. We just need

"Closure under addition and closure under scalar multiplication"

OR

"Closure under making linear combos"

Ex: $V = \mathbb{R}^2$, S is the subset of all vectors whose second entry is zero.

Closure under addition: $\begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} a+b \\ 0 \end{bmatrix}$ which is again in S.

Closure under scalar multiplication: $c \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} ca \\ 0 \end{bmatrix}$ which is again in S.