The Transpose and Its Properties

Transpose of a matrix: If $A = [a_{ij}]$, then its transpose is $A^T = [a_{ji}]$
i.e. the i^{th} row of A is the i^{th} column of A^T.
So, if A is $m \times n$, then A^T is $n \times m$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \Longrightarrow A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}, I^T = I$$

Properties of transposes:

- $(AB)^T = B^T A^T$ (note the reverse order)(this is non-trivial to prove)
- $(A^T)^{-1} = (A^{-1})^T$
  Proof: $A^T(A^{-1})^T = (A^{-1}A)^T = I^T = I$, so $(A^{-1})^T$ is the inverse of A^T.
- $(A^T)^T = A$