More on Finding Inverses

But all these linear systems have the same matrix A, and therefore have the same G-E steps! Thus we can do the following:

$$\underbrace{Ab_1 = e_1}_{[A|e_1] \xrightarrow{\text{G-J}} [I|b_1]} \qquad \underbrace{Ab_2 = e_2}_{[A|e_2] \xrightarrow{\text{G-J}} [I|b_2]} \qquad \cdots \qquad \underbrace{Ab_n = e_n}_{[A|e_n] \xrightarrow{\text{G-J}} [I|b_n]}$$

$$\underbrace{[A| \; e_1 \quad e_2 \quad \cdots \quad e_n \;] \xrightarrow{\text{G-J}} [I| \; b_1 \quad b_2 \quad \cdots \quad b_n \;]}$$

$$[A|I] \xrightarrow{\text{G-J}} [I|B] = [I|A^{-1}]$$

i.e. augment A with the identity I and perform G-J elimination. The result, if it is successful, is the identity I augmented with A^{-1}.