Finding Inverses

Let

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

denote the columns of the identity I. Then we can write $I = \begin{bmatrix} e_1 & e_2 & \cdots & e_n \end{bmatrix}$ in terms of its columns. Similarly write B in terms of its columns: $B = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}$. Then

$$\begin{aligned} AB = I \Leftrightarrow A \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} &= \begin{bmatrix} e_1 & e_2 & \cdots & e_n \end{bmatrix} \\ \Leftrightarrow \begin{bmatrix} Ab_1 & Ab_2 & \cdots & Ab_n \end{bmatrix} &= \begin{bmatrix} e_1 & e_2 & \cdots & e_n \end{bmatrix} \\ \Leftrightarrow Ab_1 = e_1, Ab_2 = e_2, \ldots, Ab_n = e_n & \end{aligned}$$

and so we have n linear systems to solve for the unknown vectors $b_1, \ldots, b_n$.