Permutation Matrices

Permutation matrices P: A permutation matrix is any matrix obtained from I by permuting some of its rows. For example

$$P = \text{Row 1} \rightarrow \text{Row 3 and Row 3} \rightarrow \text{Row 1 of } I$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Note that

$$PA = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{bmatrix}$$

that is, multiplying a matrix by a permutation matrix permutes its rows according to the exchanges for that permutation matrix.