

An Example

Here is a factorization found by G-E:

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}}_A = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}}_L \underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_U$$

Then the two triangular systems are

$$\begin{array}{l} x_1 - x_2 = c_1 \\ x_2 - x_3 = c_2 \\ x_3 - x_4 = c_3 \\ x_4 = c_4 \end{array} \quad \begin{array}{l} c_1 = 1 \\ -c_1 + c_2 = 2 \\ -c_2 + c_3 = 1 \\ -c_3 + c_4 = 3 \end{array} \quad \text{where } b = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

A quick calculation gives

$$c_1 = 1, c_2 = 3, c_3 = 4, c_4 = 7, x_4 = 7, x_3 = 11, x_2 = 14, x_1 = 15.$$