The A = LU Factorization

Similarly set

$$F^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, G^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Then

$$\begin{aligned} U = GFEA \Longrightarrow \\ G^{-1}U = G^{-1}GFEA = IFEA = FEA \\ F^{-1}G^{-1}U = F^{-1}FEA = IEA = EA \\ E^{-1}F^{-1}G^{-1}U = E^{-1}EA = IA = A \end{aligned}$$

We can do a similar calculation beginning with $c = GFEb$. Therefore

$$A = LU, b = Lc \quad \text{with} \quad L = E^{-1}F^{-1}G^{-1}$$