

Introduction to Inverses

Can we find an elementary matrix that “undoes” what $E_{ij}(a)$ does to a matrix (by multiplication)? Since $E_{ij}(a)$ adds a times row i to row j , we can undo this by immediately after applying $E_{ij}(-a)$: add $-a$ times row i to row j . Thus

$$E_{ij}(-a)E_{ij}(a) = I$$

We call $E_{ij}(-a)$ the inverse of $E_{ij}(a)$ and denote it by $E_{ij}(a)^{-1}$. Thus

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\text{since } E^{-1}E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$