Echelon Form of a Matrix

Let us look back at

$$\begin{aligned} 2u + v + w &= 5 \\ 4u - 6v + 0w &= -2 \\ -2u + 7v + 2w &= 9 \end{aligned}$$

We have defined matrix multiplication so that this sytem is $Ax = b$ where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}, b = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}, x = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

After G-E we have an equivalent system (i.e. same solutions) $Ux = c$ where

$$U = \underbrace{\begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{row Echelon form of } A}, c = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

U is called the **row Echelon form** of A (REF).