We don't have any choice but to compute AA^T and find its nullspace:

$$AA^T = \begin{bmatrix} 1 & 3 \\ -4 & -4 \\ 2 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -4 & -4 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}^T = \begin{bmatrix} 10 & -16 & -4 & 6 \\ -16 & 32 & 0 & -16 \\ -4 & 0 & 8 & 4 \\ 6 & -16 & 4 & 10 \end{bmatrix}$$

$$\Longrightarrow \xrightarrow{G-J} \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow x = \begin{bmatrix} 2x_3 + x_4 \\ x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

We need to use Gram-Scmidt to get an orthonormal basis of the nullspace. We can take (by normalization)

$$u_3 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

Then set

$$B = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} - (\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \end{bmatrix}^T \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}) \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

and normalize to get

$$u_4 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{bmatrix}$$

This means that the SVD of A is $U\Sigma V^T$ where

$$U = \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

and Σ and V as given previously.

Then the decomposition of A into a sum of rank 1 matrices is:

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T = 2\sqrt{12} \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ 0 \\ \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}^T + \sqrt{12} \begin{bmatrix} -\frac{1}{\sqrt{6}} \\ 0 \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 2 \\ -4 & -4 \\ 0 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 2 & -2 \\ 1 & -1 \end{bmatrix}$$