Non-Commutativity (again)

Example (continued): Let

$$G = E_{23}(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

This elementary matrix performs the third step in G-E on A, arranging a zero below the second pivot. This time

$$EG = \text{ Row 2 + (-2) x Row 1 of } G = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$GE = \text{ Row 3 + Row 2 of } E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

In the product EGA, G adds row 2 of A to row 3 of A (before E does its thing). In the product GEA, G adds row 2 of the modified A (modified by E) to row 3, so we should not expect the same result.