

The Algebra of Matrices

$A(BC) = (AB)C$ Multiplication is Associative

$A(B + C) = AB + AC$ Multiplication (on left) Distributes Over Addition

$(B + C)A = BA + CA$ Multiplication (on right) Distributes Over Addition

For real numbers these two distributive properties are the same. For matrices they are different because:

$AB \neq BA$ (in general) Multiplication is not Commutative

Example:

$$E = E_{12}(-2) = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F = E_{13}(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow EF = FE$$

Indeed, both EF and FE perform the first two steps in G-E on A , i.e. arranging zeros below the first pivot. The order we do this in shouldn't matter.