Matrix Multiplication

Definition:

$$\text{Row vector} \times \text{Column vector} = \left[\begin{array}{ccc} a_1 & \cdots & a_n \end{array}\right] \left[\begin{array}{c} b_1 \\ \vdots \\ b_n \end{array}\right]$$

$$= a_1 b_1 + \cdots + a_n b_n \text{ (a real number)}$$

Note that both vectors must have the same length.

Definition: If A is $m \times n$, with row vectors $a_1, ..., a_m$ (in $\mathbb{R}^n$) and B is $n \times p$ with column vectors $b_1, ..., b_p$ (in $\mathbb{R}^n$), then AB is $m \times p$ and

$$AB = \left[\begin{array}{ccc} - & a_1 & - \\ \vdots & \vdots & \vdots \\ - & a_m & - \end{array}\right] \left[\begin{array}{ccc} | & \cdots & | \\ b_1 & \cdots & b_p \\ | & \cdots & | \end{array}\right] = \left[\begin{array}{ccc} a_1 b_1 & \cdots & a_1 b_p \\ \vdots & \ddots & \vdots \\ a_m b_1 & \cdots & a_m b_p \end{array}\right]$$