Matrix Multiplication Motivation

Question: Can we define the multiplication of matrices so that our linear system can be written in the form

$$Ax = b \ ?$$

If so, then we might be able to mimic $ax = b \Longrightarrow x = \frac{b}{a}$ for real numbers. This would require that

$$Ax = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2u + v + w \\ 4u - 6v \\ -2u + 7v + 2w \end{bmatrix}$$

Clearly we must go along a row of A and down the column x, multiplying corresponding entries as we go and then add all the products together.