Example (continued)

Also

$$AA^T = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

with eigenvalues $\lambda_1 = 3$ and $\lambda_2 = 1$ and eigenvectors

$$x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

so

$$\Sigma\Sigma^T = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

It can be seen now that $\sigma_1 = \sqrt{3}, \sigma_2 = 1, \Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and easily verified that $A = U\Sigma V^T$.