

An Example

Then

$$A^T A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

with eigenvalues $\lambda_1 = 3, \lambda_2 = 1, \lambda_3 = 0$ and eigenvectors

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

so

$$\Sigma^T \Sigma = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, V = \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$