The SVF is like the SpectralTheorem

This factorization is like the spectral factorization $A = Q\Lambda Q^T$ but we no longer need to assume that A is square or that it is symmetric. Moreover, any of the diagonal entries of Λ could be negative, but the non-zero diagonal entries of Σ are all positive.

So how do we find U, V and Σ? Note that (if this factorization is true):

$$AA^T = U\Sigma V^T(U\Sigma V^T)^T = U\Sigma V^T V\Sigma^T U^T = U\left(\Sigma\Sigma^T\right)U^T$$

where $\Sigma\Sigma^T = \mathrm{diag}(\sigma_1^2, ..., \sigma_r^2, \underbrace{0, ..., 0}_{m-r})$ is $m \times m$

But AA^T is ALWAYS a symmetric matrix and this is just its spectral factorization. We conclude that:

- the columns of U are an orthonormal basis of eigenvectors of AA^T, and
- $\sigma_1^2, ..., \sigma_r^2$ are the non-zero eigenvalues of AA^T, ordered from largest to smallest.