An Example

Ex: From previous calculations we have

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$$

$$= \sqrt{3} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}^T + 1 \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}^T$$

$$= \sqrt{3} \times \frac{1}{2\sqrt{3}} \begin{bmatrix} -1 & 2 & -1 \\ 1 & -2 & 1 \end{bmatrix} + 1 \times \frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

The singular value decomposition is an important tool in digital imaging because an image (viewed as a large array of grey scale values of pixels) can be simplified by throwing away from the singular value decomposition terms whose σ_i is very small compared to the first few σ_i's (remember that they are ordered from largest to smallest).