Changing Variables in Linear Problems

Theorem: Similar matrices share the same eigenvalues
Proof:

$$\begin{aligned}\det(B-\lambda I) &= \det(M^{-1}AM-\lambda I) = \det(M^{-1}AM-\lambda M^{-1}IM)\\ &= \det(M^{-1}(A-\lambda I)M) = \det(M)\det(A-\lambda I)(\det(M))^{-1}\\ &= \det(A-\lambda I)\end{aligned}$$

Thus A and B have the same characteristic polynomial and hence the same eigenvalues.

The connection between the eigenvectors of A and B is also important:

$$By = \lambda y \Rightarrow M^{-1}AMy = \lambda y \Rightarrow A(My) = \lambda(My)$$

so $x = My$ is an eigenvector of A whenever y is an eigenvector of B.

Similarity is connected with changing variables in linear problems.