Similarity

We conclude that

$$A = \underbrace{\lambda_1 q_1 q_1^T}_{\substack{\text{projection onto}\\ q_1 \text{ and then}\\ \text{stretch by } \lambda_1}} + \cdots + \underbrace{\lambda_n q_n q_n^T}_{\substack{\text{projection onto}\\ q_n \text{ and then}\\ \text{stretch by } \lambda_n}} \Bigg\} \text{ Spectral Decomposition of } A$$

Note that each $\lambda_i q_i q_i^T$ is a rank one matrix.

The spectral factorization of a symmetric matrix (and diagonalization in general) is a special case of a similarity transformation:

Definition: Matrix B is **similar** to matrix A if there is an invertible M such that

$$B = M^{-1} A M$$

We say then that A and B are related by a **similarity transformation**.