An Example

Ex:

$$A = \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}}_{\Lambda} \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}_{Q^T}$$

Note: If q_1 and q_2 are the first and second columns of Q, then

$$Q^T q_1 = e_1, \Lambda Q^T q_1 = 2e_1, Aq_1 = Q\Lambda Q^T q_1 = 2q_1$$

$$Q^T q_2 = e_2, \Lambda Q^T q_2 = \frac{1}{2}e_2, Aq_2 = Q\Lambda Q^T q_2 = \frac{1}{2}q_2$$

This gives us a geometric insight into how A transforms the plane into itself by matrix multiplication: