

The Hermitian of a Matrix

We introduce a simple device to help one remember this new nuance:

Definition: The **Hermitian** of a complex matrix A , denoted A^H , is the transpose of the complex conjugate of A :

$$A^H = \bar{A}^T \quad (\text{so } A^H = A^T \text{ when } A \text{ is real})$$

Now we simply use H wherever we previously used T .

Definition: (Inner product and norm of complex vectors)

$$(x, y) = x^H y = \bar{x}^T y, \quad \|x\|^2 = (x, x) = x^H x = \bar{x}^T x$$

Definition: A matrix A is **Hermitian** if $A^H = A$ (“Hermitian” is the analogue for complex matrices of “symmetric” for real matrices).

Note: $\overline{AB} = \bar{A}\bar{B}$ and $(AB)^T = B^T A^T$ imply that $(AB)^H = B^H A^H$.