

Examples

Ex: Start with $Ax = b$ and change variables with $x = My$:

$$Ax = b \Leftrightarrow AMy = b \Leftrightarrow M^{-1}AMy = M^{-1}b \Leftrightarrow By = c$$

where $c = M^{-1}b$ or $b = Mc$.

Ex: Start with $du/dt = Au$, $u(0) = u_0$ and set $u = Mv$:

$$\begin{aligned} \frac{du}{dt} = Au &\Leftrightarrow \frac{d}{dt}(Mv) = A(Mv) \Leftrightarrow \frac{dv}{dt} = (M^{-1}AM)v \Leftrightarrow \frac{dv}{dt} = Bv \\ u(0) = u_0 &\Leftrightarrow Mv(0) = u_0 \Leftrightarrow v(0) = M^{-1}u_0 \Leftrightarrow v(0) = v_0 \end{aligned}$$

where $v_0 = M^{-1}u_0$ or $u_0 = Mv_0$.

The goal in these variable changes is always to change to a simpler problem. If B is diagonal, the new problem is usually super simple: several scalar problems rather than a matrix problem.