Norms and Inner Products for Complex Vectors

We have already seen examples of vectors and matrices with complex entries. Our goal here is to introduce an inner product for complex vectors, and, in terms of it, a norm or length measure (like the modulus of a complex number).

We begin by recalling

$$\text{Reals: } |a|^2 = a^2 \qquad \text{Complex : } |a|^2 = \bar{a}a$$

$$\text{Real Vectors: } \|x\|^2 = \left\| \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \right\|^2 = |x_1|^2 + \cdots + |x_n|^2 = x^T x$$

The length or norm measure here has the fundamental property that $\|x\| = 0$ implies $x = 0$. To achieve this same feature for complex vectors, we define:

$$\text{Complex Vectors: } \|x\|^2 = |x_1|^2 + \cdots + |x_n|^2 = \bar{x}_1 x_1 + \cdots + \bar{x}_n x_n = \bar{x}^T x$$