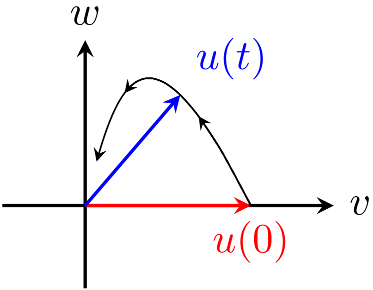
The Main Theorem

The solution is

$$u(t) = e^{tA}u(0)$$
$$= \frac{1}{2}\begin{bmatrix} e^{-t}+e^{-3t} & e^{-t}-e^{-3t} \\ e^{-t}-e^{-3t} & e^{-t}+e^{-3t} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{2}e^{-t}+\frac{1}{2}e^{-3t} \\ \frac{1}{2}e^{-t}-\frac{1}{2}e^{-3t} \end{bmatrix}$$

All solutions approach the origin due to the two decaying exponentials.



The time has come to establish

Theorem: Let A be an $n \times n$ matrix with n linearly independent eigenvectors. Then the unique solution of $\frac{du}{dt} = Au$, $u(0) = x$ is $u(t) = e^{tA}x$.