

# An Example

**Ex:** Since we have solved the diagonalization problem for  $K$ , we should be able to compute  $e^{tK}$  by the method here:

$$\begin{aligned} e^{tK} &= S \begin{bmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{bmatrix} S^{-1} \\ &= \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \cos t + i \sin t & 0 \\ 0 & \cos t - i \sin t \end{bmatrix} \frac{1}{2i} \begin{bmatrix} 1 & i \\ -1 & i \end{bmatrix} \\ &= \dots \text{ (give it a try!) } = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} \end{aligned}$$

**Ex:** Consider the following matrix and its eigenvalue analysis:

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \Rightarrow A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = (-1) \begin{bmatrix} 1 \\ 1 \end{bmatrix}, A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-3) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$