

# An Example

**Ex:** Look now at the initial value problem

$$\begin{aligned} \frac{dv}{dt} &= -w, v(0) = 1 \\ \frac{dw}{dt} &= v, w(0) = 0 \end{aligned} \iff \frac{du}{dt} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} u = Ku, u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

If our formula at the start of these notes is correct (we haven't established this yet!), the solution will be

$$\begin{aligned} u(t) &= e^{tK} u(0) \\ &= \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} \end{aligned}$$

All solutions of this system rotate about the origin.

