The Matrix Exponential

Definition: The **exponential** of a square matrix A is the square matrix

$$e^A = \exp(A) = I + A + \frac{1}{2!}A^2 + \frac{1}{3!}A^3 + \frac{1}{4!}A^4 + \cdots$$

Note: Each term in the infinite series here is an $n \times n$ matrix. When all of these are summed, we get a matrix, each of whose entries is an infinite series of real numbers. Just as the power series for e^x converges for all real x, it can be shown that the infinite series' in our matrix all converge for any matrix A. This means we have a well-defined new matrix and we will focus on how to compute it.

Ex: Let us compute $e^{\theta K}$ where

$$K = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{(a 90 degree rotation of the plane)}$$

Our first job is to compute all the powers of K.