Properties of the Matrix Exponential

Now impose the initial condition:

$$u(0) = x \Rightarrow x = Sc \Rightarrow c = S^{-1}x \Rightarrow u(t) = Se^{t\Lambda}S^{-1}x = e^{tA}x$$

and this is what we wanted to show.

Here are some properties of the exponential of a matrix:

- e^A is an invertible matrix (compare with $e^x \neq 0$)
  Proof: $\det e^A = \det(Se^{\Lambda}S^{-1}) = \det S \times \det e^{\Lambda} \times (\det S)^{-1} = \det e^{\Lambda} = e^{\lambda_1} \cdots e^{\lambda_n} \neq 0$
- If $AB = BA$, then $e^A B = Be^A$ (use the series definition of exp)
- If $AB = BA$, then $e^A e^B = e^{A+B}$ (so $e^x e^y = e^{x+y}$ does NOT apply in general to matrices)
- $e^O = I$ (the exponential of the zero matrix is the identity)
- e^{-A} is the inverse of e^A (A and $-A$ commute, so $e^A e^{-A} = e^{A-A} = e^O = I$)