

# Some Definitions and Results

**Definition:** A **dangling page** is one with no outgoing links.

**Definition:** A web is **disconnected** if you cannot get from every page to every other page (ignoring link directions).

**Definition:** A web is **strongly connected** if you can get from each page to every other page via links.

**Definition:** A matrix is **column-stochastic** if all entries are non-negative and the entries in each column sum to one.

The matrix for our example web is easily seen to be column-stochastic. For such matrices we have:

**Theorem:** Any column-stochastic matrix has  $\lambda = 1$  as an eigenvalue.

Proof: Let  $e$  be the column vector whose entries are all ones. Then the property that the columns of  $A$  sum to one is equivalent to the equation  $e^T A = e^T$  (think about it!). Transposes give us  $A^T e = e$ . Therefore  $\lambda = 1$  is an eigenvalue of  $A^T$  and hence also of  $A$ .