# A Result on Complex Eigenvalues and Eigenvectors

The preceding example has the property that the matrix's eigenvalues are complex conjugates AND the corresponding eigenvectors are complex conjugates too:

$$\lambda = i \text{ with } x = \begin{bmatrix} i \\ 1 \end{bmatrix}, \quad \bar{\lambda} = -i \quad \text{with} \quad \bar{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

Does this happen in general?

**Theorem:** If $A$ is a real matrix, then its eigenvalues and eigenvectors occur in complex conjugate pairs.

Proof: Since $A$ is real, $\bar{A} = A$, so

$$Ax = \lambda x \Rightarrow \overline{Ax} = \overline{\lambda x} \Rightarrow \bar{A}\bar{x} = \bar{\lambda}\bar{x} \Rightarrow A\bar{x} = \bar{\lambda}\bar{x}$$

and this is what we wished to show.

**Ex:** Continuing our example, we now have the factorization

$$K = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = S\Lambda S^{-1} \quad \text{where} \quad \Lambda = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, S = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}$$