Example (continued)

Case 2, $\lambda = -i$:

$$(K + iI)x = \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} ia - b = 0 \\ a + ib = 0 \end{array}$$

i times the second equation gives the first equation and in this sense $K + iI$ is singular. Thus $a = -ib$. If we set $a = -i$ we get $b = 1$ and this gives us the eigenvector

$$x = \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

The preceding calculations show that we need to deal now not only with complex numbers but also with vectors and matrices whose entries may be complex. $\mathbb{C}^n$ denotes the set of column vectors of length n whose entries are complex numbers. Similarly $\mathbb{C}^{m \times n}$ is the set of $m \times n$ matrices with complex entries. We define the conjugate of such vectors and matrices in the natural way: entry by entry.