

# Properties of Conjugates

Let us note some properties of conjugates:

- $\bar{\bar{x}} = x$  means  $x$  is real, i.e.  $a + ib = a - ib$  and this can only happen when  $b = 0$
- $\overline{x + y} = \bar{x} + \bar{y}$ . Proof:

$$\begin{aligned}\overline{(a + ib) + (A + iB)} &= \overline{(a + A) + i(b + B)} = (a + A) - i(b + B) \\ &= (a - ib) + (A - iB) = \overline{(a + ib)} + \overline{(A + iB)}\end{aligned}$$

- $\overline{\bar{x}y} = x\bar{y}$  (proof is similar)
- $\overline{\bar{x}} = x$

Finally let us give an example of division

**Ex:** The goal is to get  $i$  out of the denominator!

$$\frac{2 + i}{3 + 4i} = \frac{2 + i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} = \frac{(2 + i)(3 - 4i)}{3^2 + 4^2} = \frac{10 - 5i}{25} = \frac{2}{5} - \frac{1}{5}i$$