Modulus of a Complex Number

The modulus of a complex number measures its size. More important, we want to be sure that the complex number is zero whenever its modulus is zero. Thus we define the **modulus** of $a + ib$ as:

$$|a + ib| = \sqrt{a^2 + b^2}$$

For real numbers we know that $|a| = \sqrt{a^2}$. However this doesn't work for complex numbers because $(a + ib)^2 = a^2 + 2iab + i^2 b^2$
$= (a^2 - b^2) + 2iab$ isn't a positive real number, indeed generally it is complex rather than real. The trick is to note:

$$(a + ib)(a - ib) = a^2 - i^2 b^2 = a^2 + b^2$$

so the square root of the left side is the modulus of $a + ib$.

Definition: The **conjugate** of a complex number $x = a + ib$ is the complex number $\bar{x} = a - ib$ (just change i to $-i$). In terms of it we have

$$|x| = \sqrt{\bar{x}x}$$