Basic Manipulations of Complex Numbers

We need to define ways to manipulate complex numbers that parallel what we do with real numbers: addition, multiplication, and absolute value (size). And we need to define these so that they reduce to what we expect when the numbers we manipulate are infact real numbers. Here is addition:

$$(a + ib) + (A + iB) = (a + A) + i(b + B)$$

There are three different types of $+$ signs here. From left to right the first, third and fifth are the plus symbol appearing in the definition of a complex number. The second plus is addition of complex numbers and we are defining that here. The fourth and sixth pluses are just addition of real numbers. Note that if we just remove all the parentheses on the left side and regroup, we get the right side. It is in this sense that we want to be able to use the manipulation techniques familar from real numbers.