

An Example

Looking back at the figure we conclude that

$$a + ib = r \cos \theta + ir \sin \theta = re^{i\theta}$$

The left side here is called the **rectangular form** of a complex number while the right side is called the **polar form**.

The polar form of a complex number, since it is an exponential, allows us to use all the nice properties of exponentials when doing manipulations. For example

$$\begin{aligned} r (\cos \theta + i \sin \theta) \times s (\cos \psi + i \sin \psi) &= rse^{i\theta} e^{i\psi} = rse^{i(\theta+\psi)} \\ &= rs (\cos(\theta + \psi) + i \sin(\theta + \psi)) \end{aligned}$$

We could have expanded out the left side directly and used trig identities to turn it into the right side, but this calculation is more direct. It also illustrates how the polar coordinates of two complex numbers are connected to the polar coordinates of their product.