A Primer on Complex Numbers

So far we have not confronted an eigenvalue problem for which the eigenvalues are complex numbers. Before we do so, we give a brief introduction to complex numbers and how we work with them.

Complex numbers were introduced into mathematics so that we can be assured that any nth degree polynomial has n roots, specifically in cases such as:

$$x^2 + 4x + 5 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{4^2 - 4 \times 5}}{2} = -2 \pm \sqrt{-1}$$

In the real numbers you cannot compute the square root of a negative number, so we need to give a meaning to $i = \sqrt{-1}$.

Definition: i is a **symbol** with the property $i^2 = -1$. We call $x = a + ib$ with a and b real numbers, a **complex number**. a is called the **real part** of x and b is called the **imaginary part**. x reduces to a real number when its imaginary part b is zero. The set of complex numbers is usually denoted $\mathbb{C}$.